TEMPERATURE SIMILARITY OF HEAT TRANSFER PROCESSES

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New relationships are presented which describe the temperature and humidity of the medium at the edges of the boundary layer in heat and mass transfer processes.

The nature of the interaction between temperature conditions and heat transfer is a particularly interesting question. A good discussion of the problem is to be found in [1], where the unique role of temperature in heat transfer is emphasized: "For heat transfer phenomena temperature is the basic physical variable, since the very process of redistribution of heat in space is possible only in the presence of temperature differences. At the same time, the wide range of problems which form the basis of heat transfer studies are distinguished by the fact that the theory of similarity gives solutions for them that are independent of temperature."

The obtaining of a dimensionless group containing temperature difference in combination with the physical parameters of the medium is of definite interest in problems of both pure convective heat transfer and heat and mass transfer.

In the first case the interest is as yet purely theoretical, in that obtaining the relationship between this group and the other groups determining the thermal similarity of processes would make possible a more specific mathematical picture of the temperature variation in the boundary layer. For practical purposes the existing criterial relations required to determine heat transfer in heat-transfer apparatus, in conjunction with the well-known heat transfer equation $Q = kF\Delta t$, permit the solution, to some degree of accuracy, of the majority of heat transfer problems.

The situation is different for heat and mass transfer. A perfectly definite value of the wall temperature T_w may be obtained from the heat transfer equations $Q = kF\Delta t$ and $Q = \alpha F\theta$. For heat and mass transfer processes, however, as has been pointed out [2–4, etc.], the value of T_w is a necessary condition, not a sufficient one. This deficiency appears as a systematic deviation of the "real" process from the "theoretical" one in the direction of "humidification" of the gas in the heat and mass transfer processes occurring, for example, in condensation of vapor from a vaporized mixture or in the evaporation of liquid from a surface washed by a gaseous medium.

There is at present no scientifically based method which allows a quantitative description of this deviation. The reason for this is evidently the lack of a criterial or other relationship between the basic physical parameters of the medium and the temperature difference (and also concentration or humidity) between the medium in the flow core and directly at the heat transfer surface.

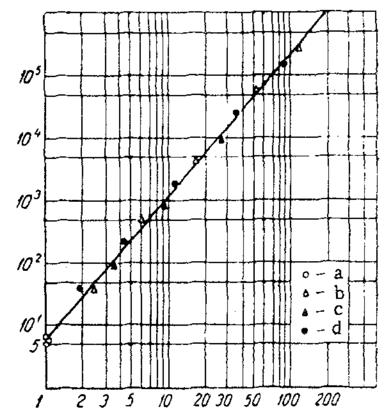

Fig. 1. Dependence of $\frac{K_\theta N_\beta^3}{\lg(T_f/T_w)}\left(\frac{l}{l_0}\right) \times 10^{31}$ (ordinate) on (γ/γ_0) (abscissa) for heat-transfer apparatus: a) according to the author's test date and the data of [2, 6, 7] (air at P ≅ ≅ 0.98 bar); b) according to test data of LenNIIkhimmash (air at P = (0.98–196) bar); c) according to test data of LenNIIkhimmash (carbon dioxide at P = (0.98–39.2) bar; d) according to calculations for gas coolers of tube-shell and "tube-in-tube" type.

The present author has obtained such a relation, which allows both the qualitative and quantitative description of the phenomenon. This was accomplished, firstly, as a result of obtaining the relation between the difference in temperatures T_f in the stream and T_w at the surface and the physical parameters of the medium (diffusivity a, specific heat c_p, specific weight γ, and thermal expansion coefficient β), followed by the extension of the general form of this relation by analogy to give the relationship of the physical parameters of the medium and the difference in humidities in the stream x_f and at the heat transfer surface x_w.

The dimensionless group containing temperature difference combined with the physical parameters of the medium was obtained from the differential equations describing heat and mass transfer in the presence of transverse mass flow [5]. The only difference from [5] consisted of the fact that the energy equation was written on the basis of conservation of energy (the first law of thermodynamics) for heat and mass transfer together, not separately. In this case the energy equation has the form

$$(\lambda + \lambda_т)\nabla^2 t + (\sigma + \sigma_т) r \nabla^2 x = \rho g \left(w_x \frac{\partial I}{\partial x} + w_y \frac{\partial I}{\partial y}\right) + \frac{AP}{\rho}\left[\left(u_v \frac{\partial \rho v}{\partial x} + v_v \frac{\partial \rho v}{\partial y}\right) + \rho_v\left(\frac{\partial u_v}{\partial x} + \frac{\partial v_v}{\partial y}\right)\right]. \quad (1)$$

From the equality of the similarity coefficients in the first term on the left side and the second term on the right side of (1), after a certain transformation, we obtain the similarity invariant K_θ, which

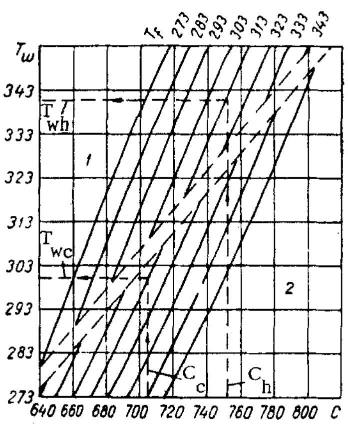

Fig. 2. Dependence of $T_w = f(T_f, C)$ for heat-transfer apparatus: 1) heating of medium; 2) cooling.

contains the temperature difference θ, and not the absolute value of the temperature, since the temperature in the terms compared comes under the derivative sign with respect to a coordinate

$$K_\theta = \frac{gl^2}{Aa^2} c_p \theta = \text{idem}. \quad (2)$$

Analogously, from the equality of the similarity coefficients of the second term on the left and the second term on the right of (1), using a similar transformation, we obtain the similarity invariant $K_{\Delta x}$ which contains the humidity difference Δx

$$K_{\Delta x} = \frac{gl^2}{Ak^2} r \Delta x = \text{idem}. \quad (3)$$

We investigated the relation between the dimensionless group (2) and the other quantities characterizing mass and heat transfer on the basis of our own experiments (on bundles of smooth and spirally ribbed tubes at atmospheric pressure of air), the data of other authors [2, 6, 7], and the data of industrial tests. It turned out, unexpectedly at first glance, that group (2), obtained from the differential equations of heat and mass transfer, gives a relationship with the parameters determining it that is valid not only for heat and mass transfer processes but also for any case of pure convective heat transfer. This requires special study. The relation obtained for K_θ was expressed in the form

$$K_\theta = C N_\beta^{-n} \lg (T_f/T_w) \left(\frac{l_0}{l}\right)\left(\frac{\gamma}{\gamma_0}\right)^m, \quad (4)$$

where

$$N_\beta = c_p/Al\beta = \text{idem}. \quad (5)$$

Here N_β is a dimensionless quantity also obtained by the method of similarity theory and characterizing the thermal expansion of the medium:

$$N_\beta = K_\theta/\text{GrPr}^2 = \text{idem}.$$

For gases the thermal expansion coefficient β at mean temperature of the medium T_f may be expressed as

$$\beta = 1/T_f.$$

Therefore, for gases

$$N_\beta = c_p T_f/Al.$$

In the group K_θ (2), θ has been taken as the difference in the arithmetic mean temperatures in the flow core and at the wall surface,

$$\theta = T_f - T_w.$$

The quantities l_0 and γ_0 are an arbitrary characteristic dimension (tube diameter) and the specific weight (air at normal conditions) taken as the zero reading. In the generalized equation (4), $l_0 = 0.01$ m, $\gamma_0 = 1.293$ kg/m^3; values of the constants are as follows: $C_1 = 0.5 \cdot 10^{31}$; n = 3; m = 2.2. These values of the constants are obtained for air at pressures from 0.98 to 196 bar, and for carbon dioxide at pressures from 0.98 to 39.2 bar, for regimes of "dry" cooling and of cooling with drying (Fig. 1).

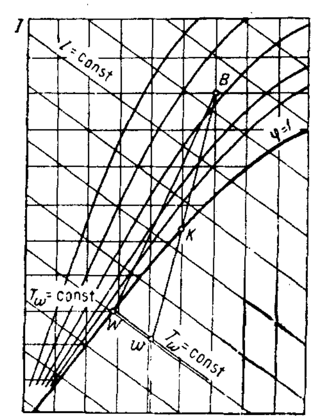

Fig. 3. Representation of a heat and mass transfer process by an x, I-diagram (moisture content-enthalpy): BW—"theoretical" process; BK—"real" process; BKw—actual process with intersection of $\varphi = 1$.

Equation (4) is reduced to the form

$$\frac{\theta}{\lg (T_f/T_w)} = C_1 \frac{Aa^2}{gl^2}\left(\frac{Al}{c_p T_f}\right)^3 \left(\frac{l_0}{l}\right) \times \left(\frac{\gamma}{\gamma_0}\right)^{2.2}, \quad (6)$$

from which the conclusion may be drawn that the difference in temperature between the medium in the flow core and at the heat transfer surface is determined by the medium properties: diffusivity, specific heat, specific weight, and thermal expansion. The relation between this difference and the hydrodynamics of the flow is determined by the logarithmic nature of the temperature variation in the boundary layer.

Equation (6) may be solved with the aid of a graph (Fig. 2) constructed according to the relation $T_W = f(T_f, C)$ for regimes of cooling (C_0) and heating (C_h):

$$C = \frac{\theta}{\lg(T_f/T_w)} = C_1 \frac{A^4 a^2 l_0}{g c_p^3 T_f^3} \left(\frac{\gamma}{\gamma_0}\right)^{2.2}, \qquad (7)$$

where C is the temperature characteristic of the heat-transfer apparatus.

For newly designed heat-transfer apparatus, analysis of the relationship between the quantities entering into (4) showed that it is regular in this case. This confirms that the existing method of calculation of heat-transfer processes in fact ensures the observance of temperature similarity.

The quantity $\theta/\lg(T_f/T_W)$ appears on the left of (6) and outwardly resembles the temperature head

$$\Delta t_{\log} = \frac{\Delta t_1 - \Delta t_2}{2.3 \lg(\Delta t_1/\Delta t_2)} = \frac{Q}{kF}.$$

However, it is not the temperature head, but may be said to be the temperature potential of the apparatus, or its temperature characteristic, dependent on the medium properties, and in contrast to Δt_{log}, independent of the heat load of the apparatus, and equally describing according to (6) both pure ("dry") heat transfer, and heat transfer accompanied by mass transfer.

In heat and mass transfer processes, however, knowledge of the mean temperature T_W of the heat transfer surface is insufficient to characterize the state of a vapor-gas mixture at the heat transfer surface. The state of a vapor-gas mixture directly at the surface may only be described by two parameters, for example, the temperature T_W and moisture content x_W. As pointed out above, the existing method of calculating heat and mass transfer processes does not supply the data required to determine the moisture content of the mixture at the heat transfer surface in the "real" process. It is precisely the lack of these data that has so far compelled us to use the hypothesis that the vapor-gas mixture at the heat transfer surface has relative humidity $\varphi = 1$, and temperature equal to that of the wall T_W ("theoretical" process). However, it has been convincingly shown by tests [2, 4, etc.] that the "real" heat and mass transfer process intersects the saturation curve at considerably higher temperatures than T_W.

Extension of relation (4) to mass transfer, using the analogy between mass transfer and heat transfer, allows us to establish the following. For mass transfer a relation of the form

$$K_{\Delta x} = C_2 \lg(x_f/x_w) N_\beta^{-n} (l_0/l)(\gamma/\gamma_0)^m, \qquad (8)$$

analogous to (4), occurs only in the case when we take the moisture content at the surface x_W (Fig. 3) to be that corresponding to the point of intersection of the process line with the isotherm T_W = const in the region of supersaturated states of the vapor-gas

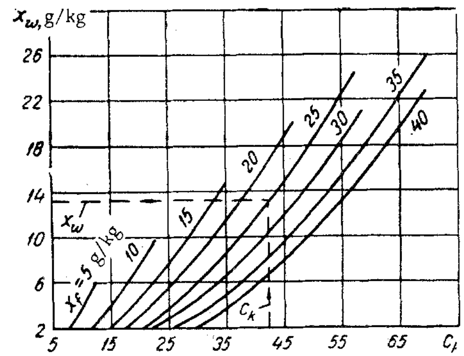

Fig. 4. The relation $x_W = f(x_f, C_k)$ for heat-transfer apparatus.

mixture. Substitution in (8) of values of the moisture content x_k corresponding to the point of intersection of the line of the "real" process with the saturation curve $\varphi = 1$ does not give a regular dependence. The test points in this case are distributed randomly over the whole graph. This indicates that the point K does not describe the state of the mixture at the heat transfer surface, but represents some intermediate state of the mixture in the boundary layer.

This conclusion supports Luikov's [8] hypothesis of "volume evaporation" of condensate in the boundary layer under conditions of forced motion of the medium, separating from the surface at temperature T_W, with consequent "humidification" of the medium.

Relation (8) was investigated quantitatively by the author for bundles of smooth tubes and tubes with spiral ribs in the temperature range T_f = 293°–313° K (20°–40° C).

The narrow range of temperature variation did not allow elucidation of the nature of the dependence of $K_{\Delta x}$ on the thermal expansion N_β of the medium. Further work in this direction is required. For the range of temperature indicated, the following relation was obtained, and may be used to calculate heat and mass transfer processes (at atmospheric pressure):

$$K_{\Delta x} = 1.14 \cdot 10^{10} \lg(x_f/x_w)(l_0/l)^2, \qquad (9)$$

whence

$$C_k = \frac{\Delta x}{\lg(x_f/x_w)} = 1.14 \cdot 10^{10} \frac{Ak^2}{g l^2 r} \left(\frac{l_0}{l}\right)^2. \qquad (10)$$

Here C_k is a quantity characterizing the drying capacity of the heat-transfer apparatus, and depends on the physical properties of the medium—the diffusion conductivity k, the latent heat of condensation r, the thermal expansion β, and the specific weight γ (in view of (10) the last two quantities are not taken into account in the range of pressure and temperature investigated).

Equation (10) is solved in a similar way to (6) with the aid of the graph $x_w = f(x_f, C_k)$ (Fig. 4).

NOTATION

A—thermal equivalent of mechanical work; g—acceleration due to gravity; c_p—specific heat of medium; T_f and T_w—arithmetic mean temperatures of medium in flow core and at heat transfer surface; w_x and w_y, u_v and v_v—projections of mixture and vapor velocities on the X and Y axes; λ and λ_T—molecular and turbulent thermal conducitvities of medium; σ and σ_T—molecular and turbulent diffusion coefficients; P—pressure; r—latent heat of condensation; l—characteristic geometrical dimension (tube diameter); l_0—arbitrary characteristic dimension taken as zero reading; x—humidity of mixture; I—heat content of mixture; ρ and ρ_v—density of mixture and vapor; γ—specific weight; a—thermal diffusivity; k—diffusion conductivity; Gr—Grashof number; Pr—Prandtl number.

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